CALCULATION OF THE NONSTATIONARY INTERACTION OF JET FLOWS WITH A TWO-DIMENSIONAL TARGET

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A finite-difference scheme is employed to calculate the flow field in the region of interaction of nonuniform jet-type flows with a flat target at normal incidence. Results obtained in the numerical solution are presented.

In recent years a number of papers have appeared in which finite-difference methods are used to solve practical problems of gasdynamics for an ideal gas. In [1, 3] studies were made of the gas flow in the gas line of a reaction-type motor and of the outflow of a supersonic jet from a nozzle; in [4, 5] studies were made of the behavior of the gas in a shock layer arising from the impingement of a uniform supersonic flow onto a blunt body. In the present paper we apply a finite-difference method to calculate the flow field in the region of interaction of a nonuniform subsonic jet-type flow with an infinitely large flat target. The flow is assumed to be two-dimensional.

We consider the nonstationary interaction of the nonuniform flow of an ideal incompressible liquid flowing out of an aperture of finite width D and impinging onto an infinitely large flat target, placed normal to the flow at a distance X (Fig. 1). We introduce a distance $x_2 = Y$, where the influence of the target on the flow can be considered to be negligibly small. The outward flow ($x_2 = Y$) is set into motion impulsively at the instant of time $t = 0$ with a speed $U_\infty = U_\infty(x_1)$ which varies in the transverse direction (x_1) and is independent of the time. In the coordinate system $x_1 0 x_2$ the fundamental dimensionless equations describing the flow may be written as follows:

$$\frac{\partial v_k}{\partial x_k} = 0, \tag{1}$$

$$\frac{\partial v_i}{\partial t} + \frac{\partial (v_i v_k)}{\partial x_k} = -\frac{\partial P}{\partial x_i}, \quad i, k = 1, 2. \tag{2}$$

In Eqs. (1) and (2) all the linear dimensions are referred to the aperture width D; the speed is referred to the magnitude of the outward flow speed U_0 on the line of symmetry ($x_1 = 0$); the pressure is referred to ρU_0^2 and the time is referred to the ratio D/U_0.

The divergence operation in Eq. (2), combined with the continuity equation (1), yields Poisson's equation; the latter is then used to calculate the pressure at each instant of time for known values of the speed:

$$\frac{\partial v_k}{\partial x_i}\frac{\partial v_i}{\partial x_k} = -\frac{\partial^2 P}{\partial x_i \partial x_i}. \tag{3}$$

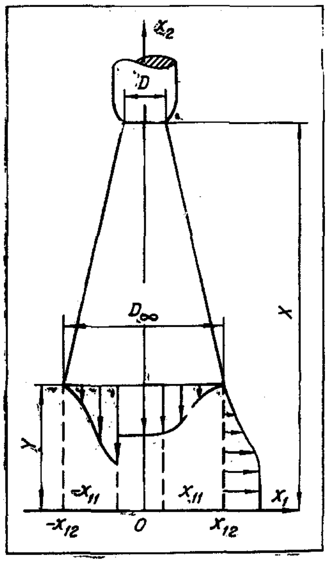

Fig. 1. Physical model and system of coordinates.

The system of Eqs. (2) and (3) is integrated in the region $F[-D_\infty/2 \le x_1 \le D_\infty/2,\ -Y \le x_2 \le Y,\ t \ge 0]$, where D_∞ is the width of the jet flow

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at the distance $x_2 = Y$ from the target. The initial and boundary conditions for the problem considered are of the form

$$t<0; \quad v_1 = v_2 = 0, \quad P = P_\infty = \text{const} \tag{4}$$

for all $(x_1, x_2) \in F$;

$$t \geqslant 0, \quad v_1 = U_{\infty 1}, \quad v_2 = U_{\infty 2}, \quad \overline{U}_\infty = U_{\infty 1}\overline{i} + U_{\infty 2}\overline{j}, \tag{5}$$
$$P = P_\infty, \quad \partial P/\partial x_1 = 0, \quad x_2 = \pm Y;$$
$$v_2 = 0, \quad x_2 = 0; \tag{6}$$
$$v_2 = 0, \quad P = P_\infty, \quad \frac{\partial P}{\partial x_2} = 0, \quad x_1 = \pm \frac{D_\infty}{2},$$
$$v_1(D_\infty/2) = v_1(-D_\infty/2). \tag{7}$$

The condition (5) is characteristic of nonuniform free jet-type flows in which the pressure gradient across the jet is negligibly small; condition (6) follows from the nonpenetrability of the target; the condition (7) is based on the assumption that the flow at the distances $x_1 = \pm D_\infty/2$ is symmetric relative to $x_1 = 0$ and is completely reconstituted in a direction along the surface of the target. The latter is characteristic for a jet adjacent to a wall.

For the difference approximation of the differential equations (2) and (3) we introduce steps Δt, Δx_1, Δx_2 in the difference mesh along the $0t$, $0x_1$, $0x_2$ axes, respectively. We denote flow parameters at the point $M(i, j)$ at the time instant t by $A^{i,j,t}$ and those at the point $N(i \pm \Delta x_1, j \pm \Delta x_2)$ at the time $t + \Delta t$ by $A^{i\pm 1, j\pm 1, t+1}$. Henceforth we omit t from the notation, writing $A^{i,j}$ in place of $A^{i,j,t}$. Using central differences for spatial derivatives and forward differences for time derivatives, we write the finite-difference analog of Eqs. (2) as follows:

$$\frac{2\Delta x_1 \Delta x_2}{\Delta t} v_1^{i,j,t+1} = -\Delta x_2 v_1^{i+1,j} v_1^{i+1,j} + \Delta x_2 v_1^{i-1,j} v_1^{i-1,j}$$
$$-\Delta x_1 v_2^{i,j+1} v_1^{i,j+1} + \Delta x_1 v_2^{i,j-1} v_1^{i,j-1} - \Delta x_2 (P^{i+1,j} - P^{i-1,j}) + \frac{2\Delta x_1 \Delta x_2}{\Delta t} v_1^{i,j}. \tag{8}$$

An analogous expression can also be written for the v_2 component of the velocity. For Eq. (3) we obtain

$$-4(P^{i+1,j} + P^{i-1,j} + P^{i,j+1} + P^{i,j-1} - 4P^{i,j})$$
$$= (v_1^{i+1,j} - v_1^{i-1,j})^2 + \left(\frac{\Delta x_1}{\Delta x_2}\right)^2 (v_2^{i,j+1} - v_2^{i,j-1})^2 + 2\frac{\Delta x_1}{\Delta x_2}(v_1^{i,j+1} - v_1^{i,j-1})(v_2^{i+1,j} - v_2^{i-1,j}). \tag{9}$$

The difference equations are solved in the following order.

1. At the time instant $t = 0$ we take $v_1 = v_2 = 0$ at all the nodes of the mesh with the exception of the boundary $x_2 = Y$. For the case in which an approximate solution of the problem in a stationary formulation is known throughout the whole region F, it is convenient to use the data of this solution to establish the acceleration.

2. For the given velocity values at time $t = 0$ we solve the linear system of equations for the pressure at the mesh modes, obtaining it from Eq. (9) for each node of the mesh. By fixing the pressure at the nodes $x_1 = \pm D_\infty/2$, we make the system nonsingular. An iterational process is used as long as the difference in the quantities P in consecutive iterations at a number of selected nodes stays larger than 10^{-6} at all of these nodes simultaneously. To accelerate convergence of the iterational process we use the method of overrelaxation. Equation (9) is replaced by an equation of the form

$$P_{(n+1)}^{i,j} = P_{(n)}^{i,j} + k\left\{\frac{1}{4}(P_*^{i+1,j} + P_*^{i-1,j} + P_*^{i,j+1} + P_*^{i,j-1})\right.$$
$$\left. + \frac{1}{16}\left[(v_1^{i+1,j} - v_1^{i-1,j})^2 + \left(\frac{\Delta x_1}{\Delta x_2}\right)^2 (v_2^{i,j+1} - v_2^{i,j-1})^2 + 2\frac{\Delta x_1}{\Delta x_2}(v_1^{i,j+1} - v_1^{i,j-1})(v_2^{i+1,j} - v_2^{i-1,j})\right] - P_{(n)}^{i,j}\right\}, \tag{10}$$

where k is the coefficient of overrelaxation; the subscripts (n) and (n + 1) denote values for the n-th and (n + 1)-th iterations; the subscript * denotes the next to the last correctional quantity. As the result of numerical experiments we chose the overrelaxation coefficient equal to k = 1.7.

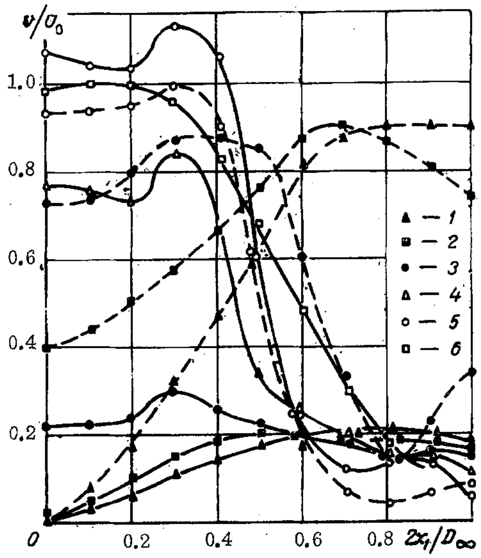

Fig. 2. Velocity profiles in the jet-target interaction region ($\xi = 1.0$) at various times t: solid line, $t = 0.7$; dashed line, $t = 3.0$. Data points corresponding to the enumeration 1, 2, 3, 4, 5, 6 correspond to x_2/Y values of 0, 0.2, 0.4, 0.6, 0.8, 1.0, respectively.

If we denote the difference in the value of the pressures in the n-th and (n−1)-th iterations of each of the mesh modes by $e_{(n)} = P_{(n)} - P_{(n-1)}$, then, for the value of k chosen, a value of the ratio $\|e_{(n+1)}\|/\|e_{(n)}\|$ equal to 0.999 is attained at the initial instant when the number of iterations $n < 200$. It should be noted that when values of the velocity in the computational region F close to the stationary solution are used for the initial conditions at $t = 0$ the time for the iterational process is considerably shortened. When the law for velocity propagation at the initial instant is well chosen, the number of iterations does not exceed $n = 20$.

3. Knowing the velocity components and the pressure at each node point in the computational region F at the time instant $t = 0$, we obtain the values of the velocity at an instant of time Δt seconds later by using Eq. (8) for v_1 and the equation analogous to it for v_2. The pressure is then calculated from Eq. (9) using the known values of the velocity at the time instant Δt.

4. The process is repeated at succeeding time instants.

Establishment of the flow in time was verified by satisfying the continuity equation. This latter equation, written in difference form with central differences employed for derivatives with respect to the coordinates, has the form

$$E = v_1^{i+1,j} - v_1^{i-1,j} + v_2^{i,j+1} - v_2^{i,j-1}, \quad E \to 0. \tag{11}$$

In calculating boundary nodes of the region in a coordinate direction where central differences cannot be evaluated, we used either forward or backward differences.

As an example of a nonuniform flow impinging on a target, we consider the flow (Fig. 1) consisting of a central uniform flow of speed U_0 and width x_{11}, and a peripheral flow of boundary-layer type for a two-dimensional jet with zero velocity at its outer edge (x_{12}). The specific boundary conditions for the flow impinging on the target ($x_2 = Y$) are of the form:

for the central flow,

$$U_{\infty 1} = 0, \quad U_{\infty 2} = -U_0 = \text{const}, \quad 0 \leqslant x_1 \leqslant x_{11}, \tag{12}$$

for the peripheral flow of boundary layer type for a two-dimensional turbulent jet [6]

$$U_{\infty 1} = \xi U_0 \left[\varphi \Phi'(\varphi) - \Phi\right], \quad U_{\infty 2} = \xi U_0 \Phi'(\varphi), \; x_{11} < x_1 \leqslant x_{12} = D_\infty/2, \tag{13}$$

where

$$\Phi(\varphi) = C_1 \exp(-\varphi) + \left(C_2 \cos\frac{\sqrt{3}}{2}\varphi + C_3 \sin\frac{\sqrt{3}}{2}\varphi\right) \exp\frac{\varphi}{2};$$

$$\varphi = (2c^2)^{-\frac{1}{3}} \frac{\frac{D}{2} - x_1}{X - Y}; \quad C_1 = -0.0176; \quad C_2 = 0.1337;$$

$$C_3 = 0.6876; \quad x_{11} = \frac{D}{2} - 0.981\sqrt[3]{2c^2}(X - Y);$$

$$x_{12} = \frac{D_\infty}{2} = \frac{D}{2} + 2.04\sqrt[3]{2c^2}(X - Y); \quad \sqrt[3]{2c^2} \simeq 0.09.$$

In formulating the boundary conditions for the region $-D_\infty/2 \leq x_1 \leq 0$, we used the condition of flow symmetry at the boundary $x_2 = Y$ relative to the line of symmetry $x_1 = 0$.

In Eq. (13) the coefficient ξ characterizes the ratio of the maximum speed U_m in the boundary layer to the speed U_0 of the central flow. For the case in which $U_m = U_0$, i.e., $\xi = 1.0$, the nonuniform

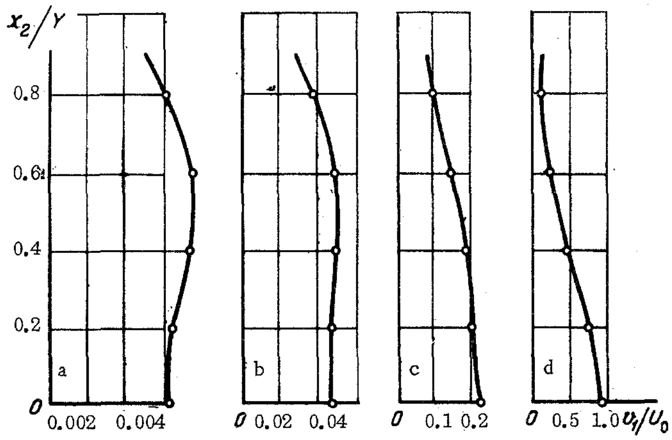

Fig. 3. Development of the velocity profile with time at the section $x_1 = \pm D_\infty/2$ for a jet with $\xi = 1.0$: curves a, b, and c are drawn for the times t = 0.05, 0.3, and 3.0, respectively.

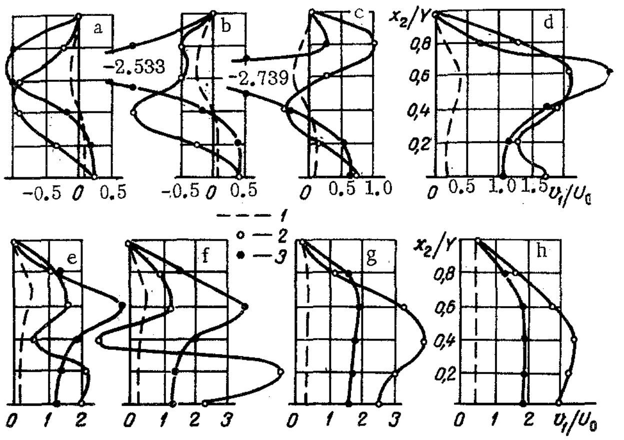

Fig. 4

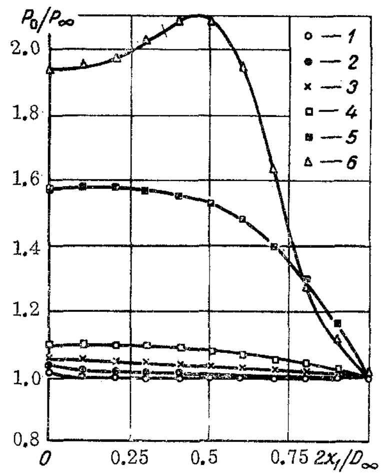

Fig. 5

Fig. 4. Profiles of tangential velocity component in the jet–target interaction region: for the data curves 1 and 2, $\xi = 2.4268$ and the time t = 0.25 and 0.625, respectively; for the data curve 3, $\xi = 7.2805$ and t = 0.15. For the individual graphical plots a, b, c, d, e, f, g, and h the value of $2x_1/D_\infty$ used was 0.1, 0.2, 0.25, 0.4, 0.45, 0.5, 0.8, and 0.95, respectively.

Fig. 5. Pressure distribution along the target at various times t for a jet with $\xi = 2.4268$: Data curves 1, 2, 3, 4, 5, and 6 are for t = 0.25, 0.45, 0.5, 0.55, 0.6, and 0.615, respectively.

flow represents a subsonic jet impinging on the target within the limits of its initial portion. When $\xi > 1.0$, the flow, formed in accordance with our assumptions, is the analog of the flow behind a curvilinear shock which forms in front of a target due to the action on it of a supersonic underexpanded jet [7].

Upon choosing the initial conditions for the subsonic jet interacting with the target within the limits of the initial portion of the jet ($\xi = 1.0$), we determined the initial velocity distribution by solving approximately the problem concerning impingement of the jet on the target in a stationary formulation [8]. For an ideal gas the approximate solution reduces to the following distribution of the velocity components along the target:

$$v_1 = \frac{U_0}{Y} \Phi_1 (x_1), \qquad v_2 = -\frac{U_0}{Y} \Phi_1' (x_1), \tag{14}$$

where

$$\Phi_1'(x_1) = \begin{cases} 1, & 0 \leqslant x_1 \leqslant x_{11}, \\ \Phi'(\varphi), & x_{11} \leqslant x \leqslant x_{12} = D_\infty/2. \end{cases}$$

In our calculations we used a difference mesh of dimension 40×10 and 40×5, with step size $\Delta x_1 = 0.05$, $\Delta x_2 = 0.1$ and 0.2, respectively, for each of the meshes, and a time step $\Delta t = 0.005$. We found that a difference mesh of size 40×5 yields flow parameters which differ insignificantly from those obtained with a mesh of twice the number of nodes. In the calculations the coefficient $\xi = 1.0$; 2.4268; and 7.2805. The quantity Y, on which the experiments with the interaction of subsonic jets ($\xi = 1.0$) with a two-dimensional target were based [9], was taken equal to D. We also fixed the data concerning the pressure distribution (P_0) over the target and that relating to the velocity component profiles v_1 and v_2 in the flow interaction region F with the target. The following initial data was used in the computations: $U_0 = 20$ m/sec; $D = 10^{-2}$ m; $P_\infty = 1$ atm; $x = 3D$.

In Fig. 2 we display calculated values of the total velocity of the flow in the subsonic jet–target interaction region ($\xi = 0.1$) at the separate times $t = 0.7$ and $t = 3.0$ (corresponding to 140 and 600 time steps). The nature of the velocity distribution over the target, for the time instants considered, stays the same on the whole; however, quantitatively the results differ substantially. At the time $t = 3.0$ the flow may be considered to be practically reconstituted in the direction along the target; the velocity profile at the exit sections $x_1 = \pm D_\infty/2$ of the interaction region calls to mind the velocity profile in an ideal jet close to a wall (Fig. 3); the magnitude of the velocity gradient in a neighborhood of the deceleration point ($x_2 = 0$) is approximately equal to U_0/D, which agrees with the data given in a number of experimental papers dealing with the study of the interaction of subsonic jets with flat targets placed normal to their flow. The error in satisfying the continuity Eq. (11) at $t = 3.0$ is equal to $E \simeq 0.1$. Hence the time instant $t = 3.0$ was taken as the final one. It should be noted that the number of iterations made in calculating the pressure in a given case appeared to be approximately constant and less than the initial number of iterations ($t = 0$) up to the time $t = 3.0$, the increase in the number of iterations with time being insignificant.

We now consider the results obtained in calculating the interaction of a composite flow with the target ($\xi > 1.0$). In Fig. 4 we present profiles of the velocity component tangent to the target ($\xi = 2.4268$ and $\xi = 7.2805$ at the time instants $t = 0.625$ and 0.15, respectively) and we also show the evolution with time of the flow along the target (for $\xi = 2.4268$). The type of nonuniform external flow considered leads to the appearance of a complex vortical flow near the target with zones of reverse flow towards the target center ($x_1 = x_2 = 0$). The vortex zones for the time instants considered are not stationary but change shape as they are displaced along the target. In Fig. 5 we present curves showing the change in the pressure P_0 along the target at various times for $\xi = 2.4268$. As can be seen from the figure, the pressure distribution for times $t \geq 0.6$ are characterized by a peripheral maximum pressure. As was noted in the experimental paper [6] for the case of the interaction of a supersonic jet with a target, the flow close to the target, with a peripheral maximum static pressure on the target, has a tendency to become unstable. With the fixed boundary $x_2 = Y$, part of the liquid accumulates in the central region since it cannot overcome the peripheral maximum pressure. From the point of view of numerical computation, the boundary conditions at the boundary $x_2 = Y$, which in this case are constant, are incorrectly posed. It should be assumed that increasing the distance Y, instead of keeping it constant and equal to D as in our analysis, will lead to an increase in the limiting time attainable in using a computational mesh. A decrease in the ratio (ξ) of the value of the maximum speed U_m at the periphery to the speed U_0 in the central region has the same effect; when $\xi = 2.4268$, the final computational time $t = 0.625$, whereas when $\xi = 7.2805$, $t = 0.15$. A time shift, even one with $\Delta t = 0.025$, leads to a loss in computational stability for the computational mesh used.

The calculations were made on a Minsk-2 computer in the Bombay Technological Institute and partly, using a standard program on the CGD-3600, at the Bombay Institute of Fundamental Research (India).

NOTATION

$x_1 0 x_2$	is the coordinate system;
F	is the region of jet and wall interaction;
D	is the slit dimension;
X	is the distance from slit cut-off to wall;
$Y(D_\infty)$	is the dimension of interaction region along normal to wall (over wall surface);

x_{11}	is the dimension of central region of constant velocity at the boundary $x_2 = Y$;
$x_{12} = D_\infty /2$;	
t	is the time;
v_1, v_2	are the velocity components along the axes $0x_1$, $0x_2$, respectively;
$\overline{v} = v_1\overline{i} + v_2\overline{j}$	is the velocity vector;
P	is the pressure;
ρ	is the density;
$U_{\infty 1}$, $U_{\infty 2}$	are the velocity components at boundary $x_2 = Y$ along the axes $0x_1$, $0x_2$, respectively;
U_0	is the velocity within central region at boundary $x_2 = Y$;
U_m	is the velocity maximum at periphery of the region of interaction;
$\xi = U_m/U_0$;	
P_∞, P_0	are the ambient pressure and the pressure at the wall, respectively;
c	is the jet turbulence constant;
Δt, Δx_1, Δx_2	are the difference grid pitches along axes $0t$, $0x_1$, $0x_2$, respectively;
E	is the error in continuity equation fulfilment.

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